# The use of a Monte Carlo method for uncertainty calculation, with an application to the measurement of neutron ambient dose equivalent rate

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# Scope

Propagation of distributions

GUM uncertainty framework

Monte Carlo method

Application: neutron ambient dose equivalent measurement

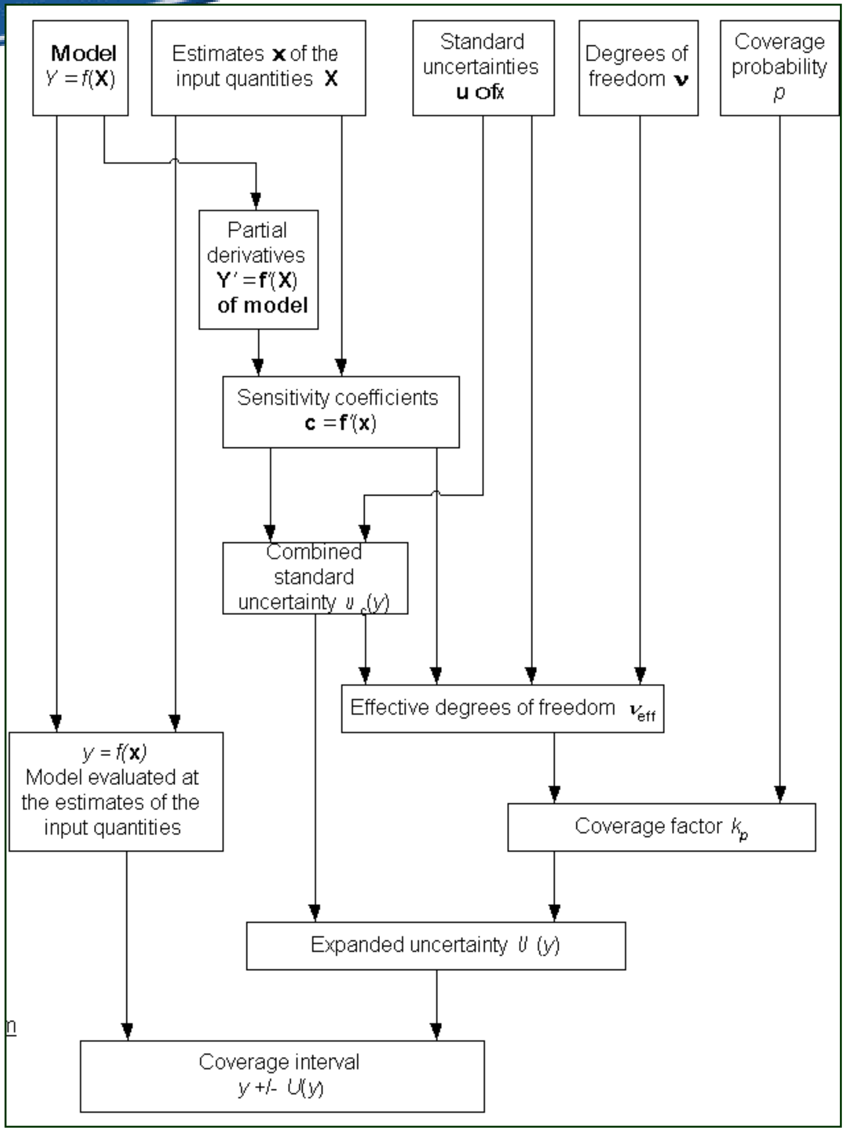
Model, input quantities, best estimates, uncertainties, PDFs

Field-specific correction factor: choice of PDF

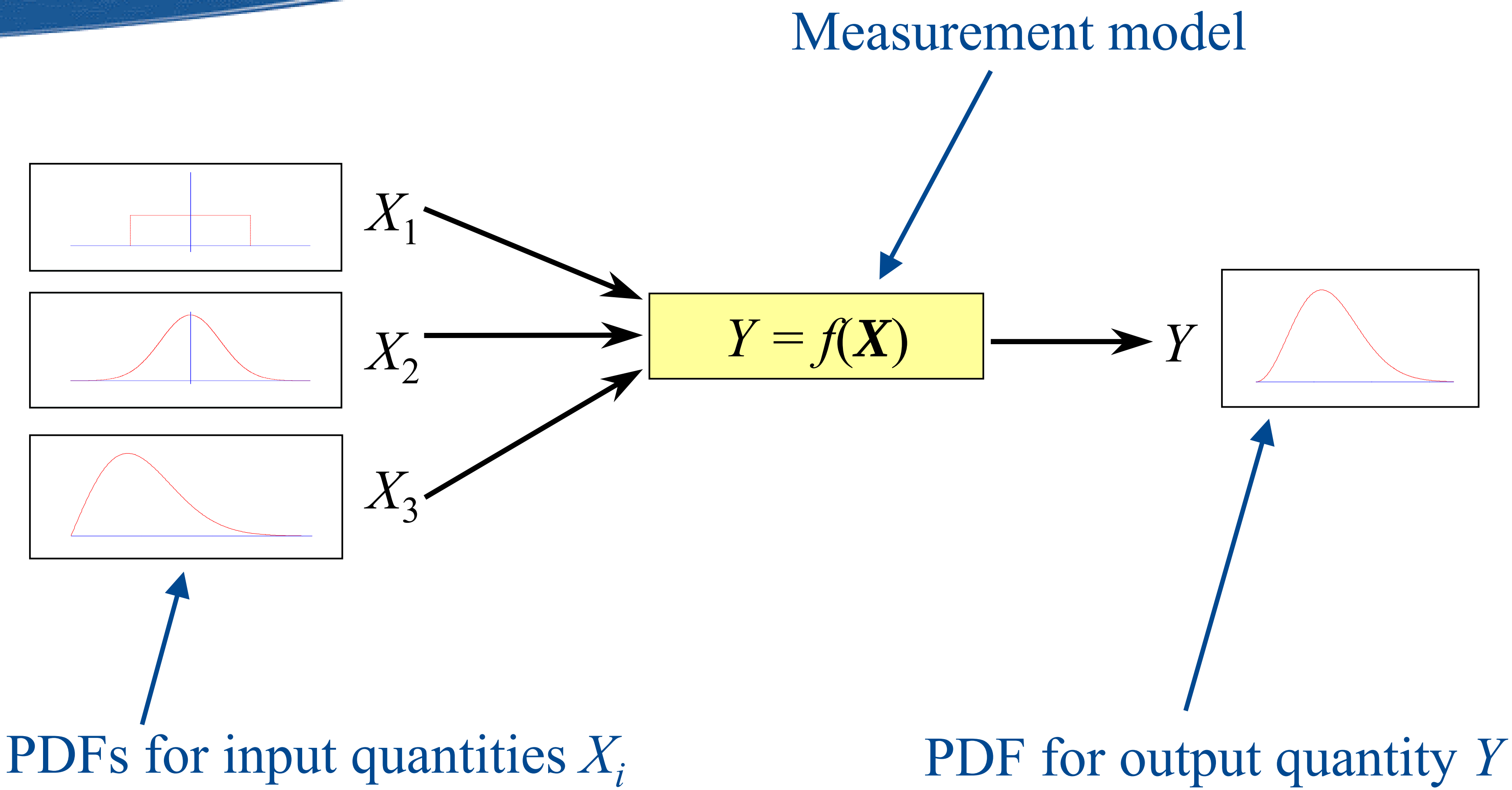
Uncertainty calculation

Conclusion

# GUM uncertainty framework



# Propagation of distributions



# Monte Carlo method (MCM)

Numerical procedure to propagate distributions through model

Based on repeated sampling from the PDFs for input quantities and evaluation of measurement model

Primary output: (approximation to) PDF for output quantity

# Features of MCM

Broad applicability: makes no assumption about

- model, e.g., linear or only mildly non-linear
- input quantities, e.g., that no one dominates
- output quantity, e.g., that it is described by a Gaussian PDF

Numerical accuracy of PDF for output quantity controlled by number of Monte Carlo trials

# Application: neutron ambient dose equivalent measurement

Measurement of neutron ambient dose equivalent rate *M*

Use of area survey instrument in workplace environment

# Model for adjusted dose equivalent rate *R* and input quantities

$$R = \frac{MBFh}{4\pi r^2 M_C K}$$

| Symbol | Input quantity |
|---|---|
| $M$ | Measured neutron dose equivalent rate |
| $B$ | Source emission rate |
| $F$ | Source emission anisotropy correction factor |
| $h$ | Fluence rate conversion coefficient |
| $r$ | Source-detector distance |
| $M_C$ | Calibration field measured neutron dose rate |
| $K$ | Field-specific correction factor |

# Best estimates of input quantities and assigned PDFs

| Quantity | Unit | Best estimate | Standard uncertainty | Assigned PDF |
|---|---|---|---|---|
| $M$ | $\mu Svh^{-1}$ | 20.0 | 5.0 % | G |
| $B$ | $h^{-1}$ | 720.0 | 1.0 % | G |
| $F$ | 1 | 1.020 | 0.5 % | G |
| $h$ | $\mu Svcm^2$ | 385.0 | 1.0 % | G |
| $r$ | cm | 150.0 | 0.3 % | G |
| $M_C$ | $\mu Svh^{-1}$ | 90.0 | 2.0 % | G |
| $K$ | 1 | ? | ? | ? |

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# Field-specific correction factor *K*

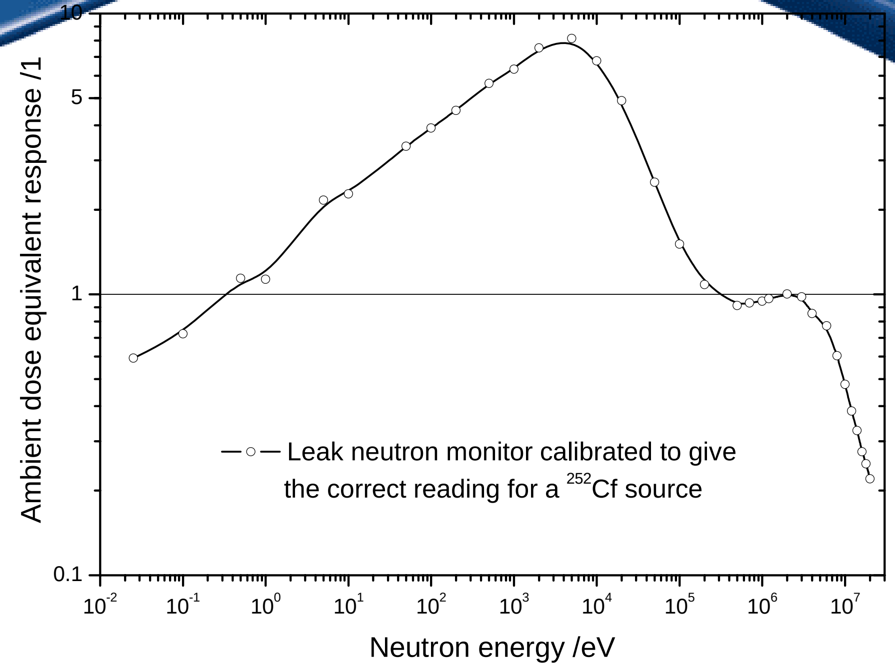
Correction factor relating to

- Use of instrument in neutron field having different energy spectrum from that in which calibrated

- Instrument not giving correct indication for dose equivalent rate as function of energy

# Field-specific correction factor *K*

- Ideal instrument has response curve with constant value unity
- Typical instrument (Leake) generally over- or under-responds in workplace field
- Degree of over- or under-response depends on how much of the dose equivalent occurs in regions where the instrument performs well compared with elsewhere




Ambient dose equivalent response of Leake neutron survey instrument

- Response reasonably good near 1 eV and $10^6$ eV
- Large over-response in intermediate energy region

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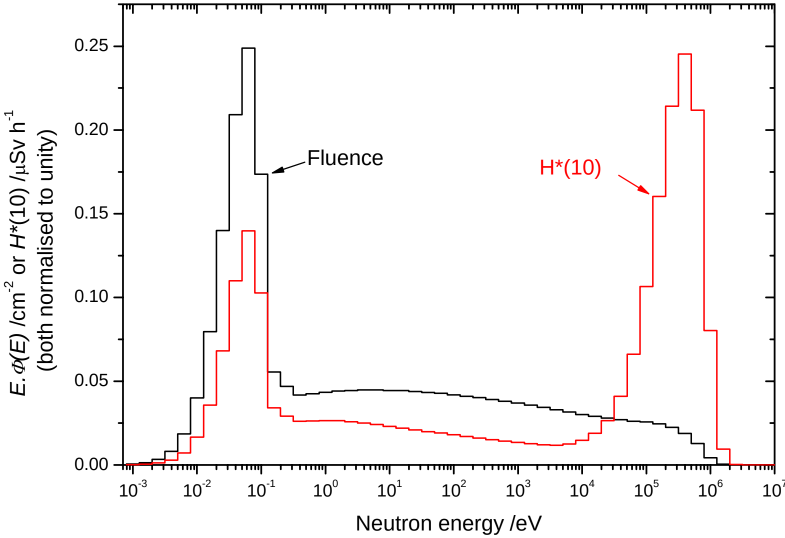
# Field-specific correction factor *K*

No instrument available to measure directly dose equivalent as function of energy

Hence derive by first measuring neutron fluence spectrum and converting to dose equivalent spectrum

# Field-specific correction factor *K*

- Conversion to ambient dose equivalent performed by multiplying fluence spectrum by fluence to ambient dose equivalent conversion coefficients
- Coefficients available as a function of neutron energy [ICRU 57]
- Approximately constant up to $10^4$ eV
- Then increase by factor of 40 up to $10^6$ eV




Fluence and ambient dose equivalent energy distributions for a UK Magnox reactor

# Field-specific correction factor *K*

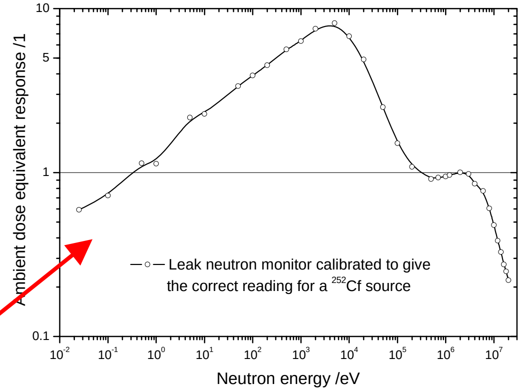
- Considerably larger values of conversion coefficients at higher energies mean that the relatively small neutron fluence at these energies produces significant contribution to total dose equivalent
- Much of contribution occurs in energy region where Leake instrument performs well
- Sufficiently large fraction outside this region
- Field-specific correction factors introduced to overcome

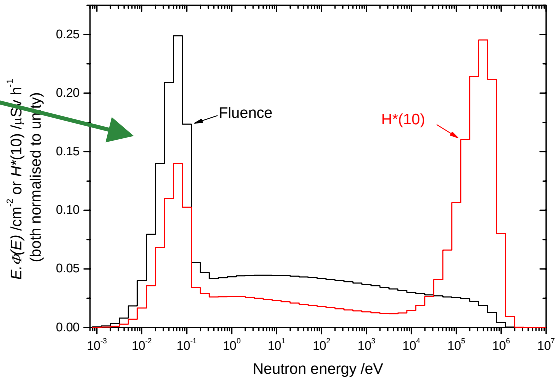
# Field-specific correction factor *K*

Each such factor obtained by deriving from spectrum a best estimate of ambient dose equivalent for a location, and comparing it with predicted response

Predicted response given by folding instrument ambient dose equivalent response with ambient dose equivalent spectrum

(or instrument fluence response with fluence spectrum)



# Field-specific correction factor *K*

In vast majority of cases, correction factors not available for field in which measurement made

Factors for small number of fields (from possibly hundreds) of similar type typically available

Consider use of this rather sparse data

Particularly, evaluation of uncertainty associated with estimate of factor

# *K*-values for Leake instrument for nuclear reactor environments

| Reactor | $H^*(10)/\Phi$ | $K$ |
|---|---|---|
| Hinkley A | 14.3 | 1.203 |
| Dungeness A1 | 15.8 | 2.134 |
| Dungeness A2 | 21.4 | 1.950 |
| Dungeness A3 | 22.0 | 1.982 |
| Dungeness A4 | 18.4 | 1.731 |
| CalderHall 1 | 14.3 | 1.357 |
| CalderHall 2 | 13.1 | 1.274 |

| Reactor | $H^*(10)/\Phi$ | $K$ |
|---|---|---|
| Trawsfyndd | 21.1 | 1.667 |
| Gosgen 1 | 44.2 | 1.232 |
| Gosgen 2 | 30.0 | 1.330 |
| Ringhals A | 28.7 | 1.724 |
| Ringhals F | 30.0 | 1.717 |
| Ringhals G | 19.3 | 1.678 |
| Ringhals L | 38.1 | 1.737 |

# Data values of *K*

Small set of independently obtained values

$$k_1, \ldots, k_n$$

each relating to field similar to that in which measurement required

Uncertainties small relative to their standard deviation

# Difficulty

Each *k*-value regarded as realization of one of set of possible values of *K* for all fields similar to that of concern

Set will have some distribution, full knowledge of which would require measurement in all fields

# Approach

1. Examine data and hypothesize hopefully appropriate distribution $G$ (normal, rectangular, …) for $K$
2. Test suitability of $G$
3. Should test not reject hypothesis, take
   a) Expectation of $K$ as characterized by $G$ as best estimate $k$ for field of concern
   b) Corresponding standard deviation as $u(k)$
4. Should test reject hypothesis, reconsider $G$

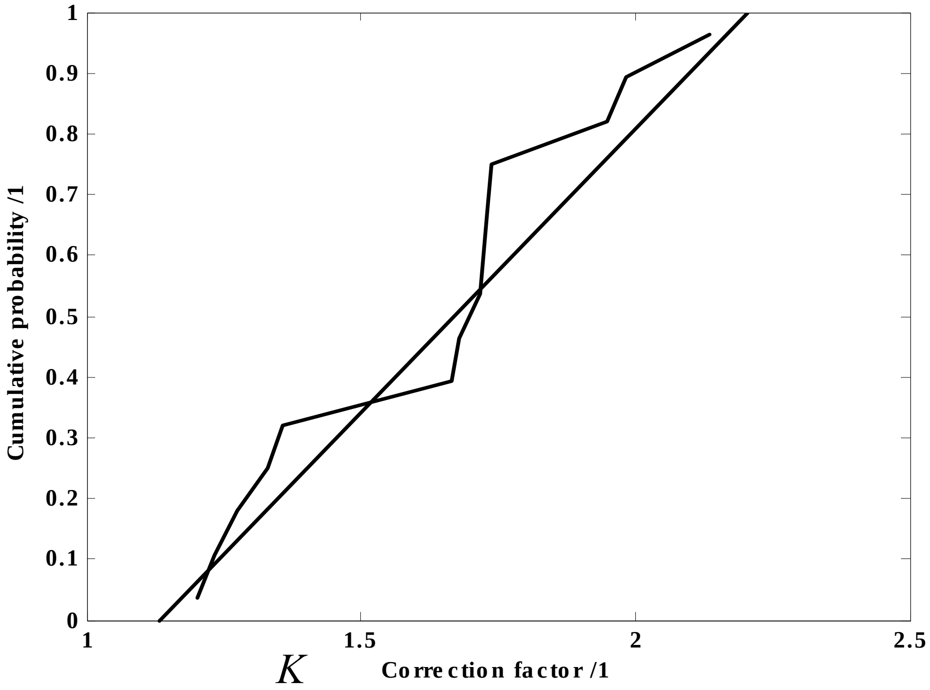
# Examine data

Cumulative frequency plot

Cumulative probability vs. ordered *k*-values

By inspection, not appreciable departure from straight line (for small sample)

(Normal distribution would give an S-shaped curve)



# Hypothesize distribution

Hypothesize rectangular distribution $R$
Estimate its parameters
Only the data $K$-values available for this purpose

Best estimates of midpoint $k_{\mathrm{M}}$ and semi-width $k_{\mathrm{W}}$ of rectangular distribution

$$k_{\mathrm{M}} = (\min k_i + \max k_i)/2$$
$$k_{\mathrm{W}} = C(\max k_i - \min k_i)/2$$
$$C = (n + 1)/(n - 1)$$ Inflation factor

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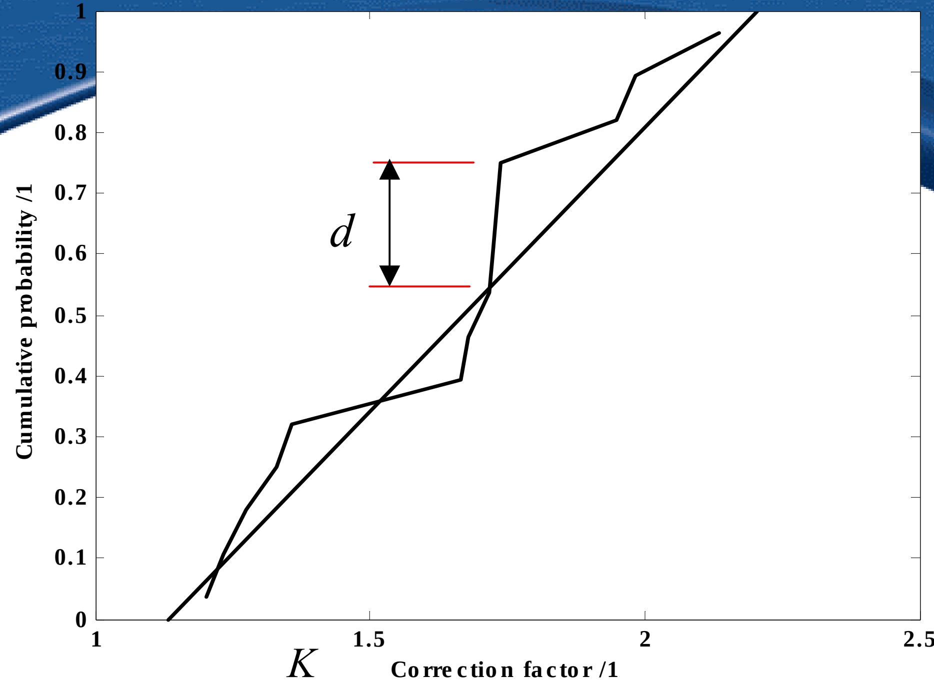
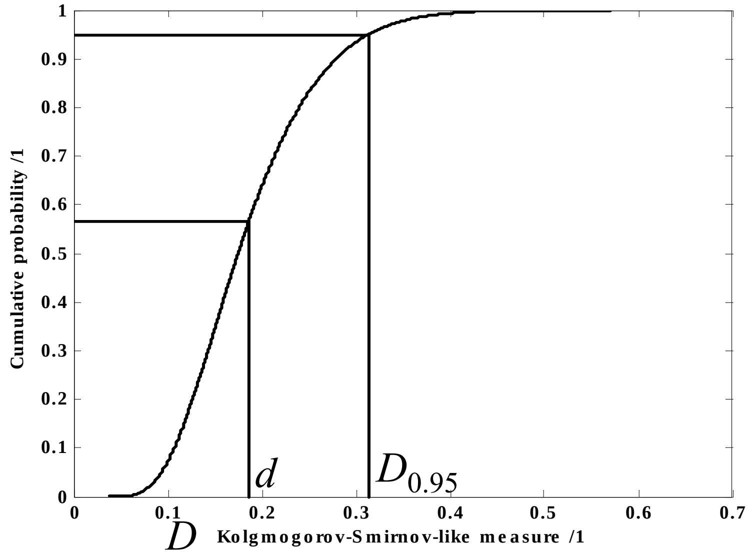
# Test hypothesis




Is $R$ consistent with the data distribution?

Measure maximum distance $d$ between the distributions

Test by estimating distribution $D$ of which $d$ is a realization:
take repeated samples of size $n$ from $R$, forming an estimate of $D$ in each case and compare with $d$

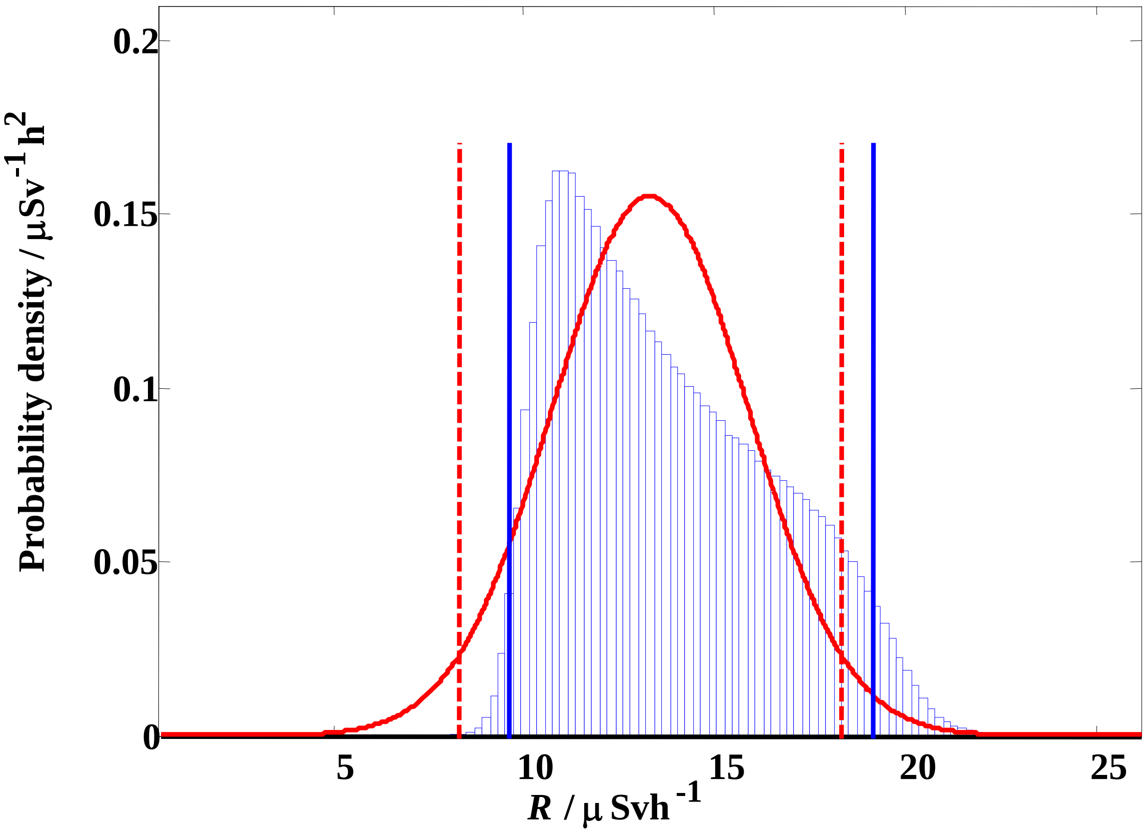



No reason to doubt hypothesis

# Completed set of input quantities

| Quantity | Unit | Best estimate | Standard uncertainty | Assigned PDF |
|---|---|---|---|---|
| $M$ | $\mu Svh^{-1}$ | 20.0 | 5.0 % | G |
| $B$ | $h^{-1}$ | 720.0 | 1.0 % | G |
| $F$ | 1 | 1.020 | 0.5 % | G |
| $h$ | $\mu Svcm^2$ | 385.0 | 1.0 % | G |
| $r$ | cm | 150.0 | 0.3 % | G |
| $M_C$ | $\mu Svh^{-1}$ | 90.0 | 2.0 % | G |
| $K$ | 1 | 1.668 | 18.6 % | R |

# PDFs for *R* from GUM uncertainty framework and MCM



# Insight via approximation

Consider influence of $K$ alone

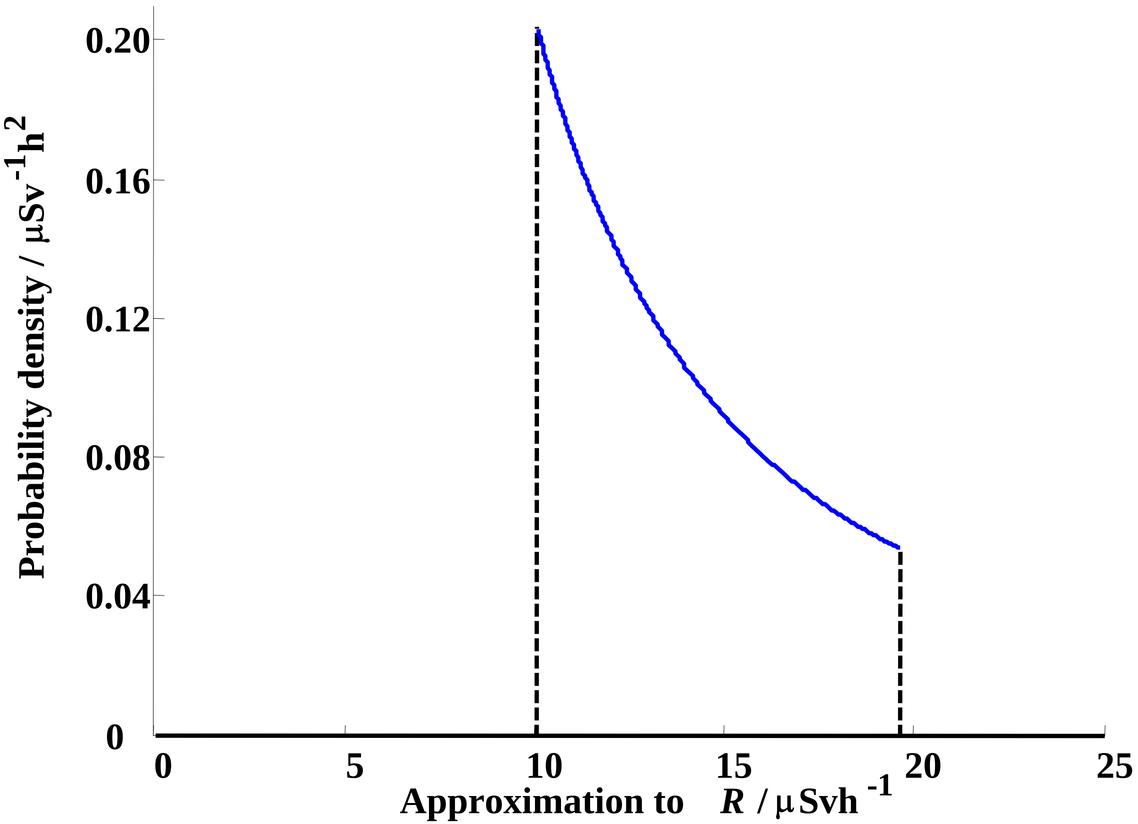
Express model as $R = \dfrac{Q}{K}, \quad Q = \dfrac{MBFh}{4\pi r^2 M_\mathrm{C}}$

Replace $Q$ by constant $q$ given by evaluating $Q$ at best estimates of relevant input quantities

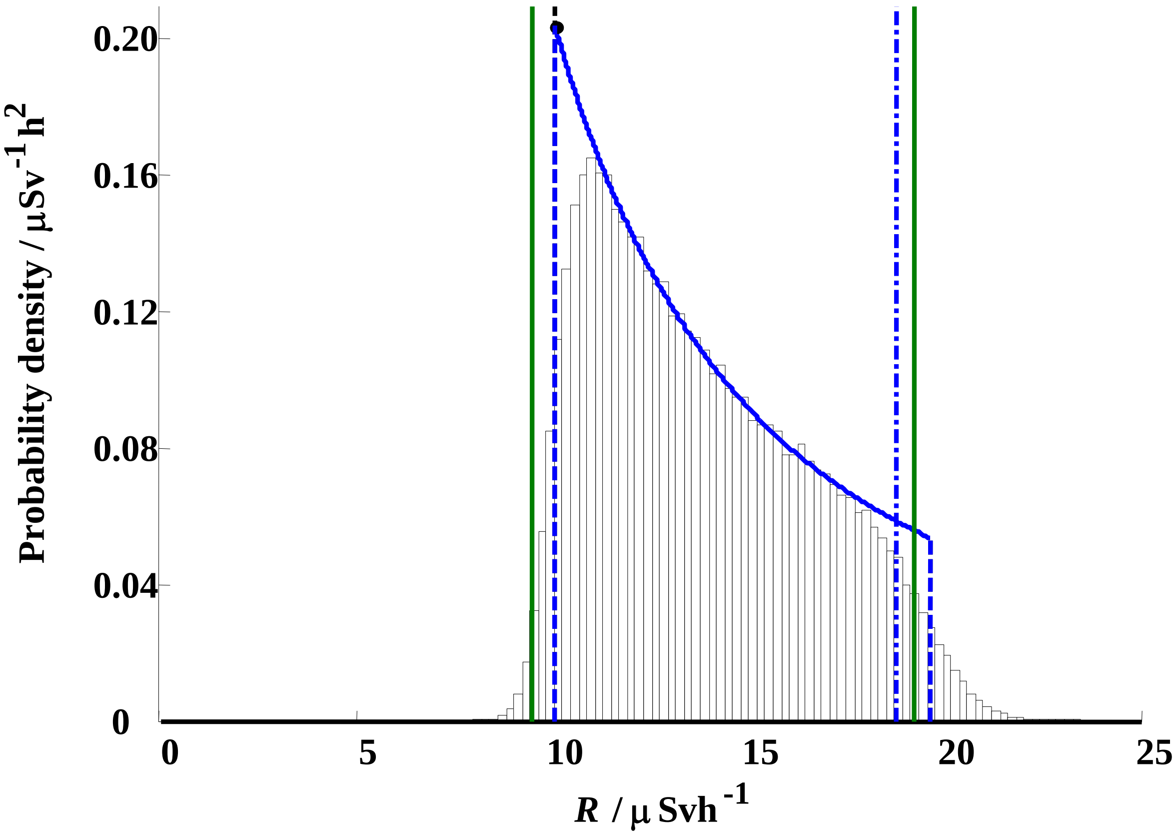
Model $\tilde{R} = \dfrac{Q}{K}$ constitutes approximation to $R$

PDF obtained analytically: inverse square

# PDF for approximation to *R*



# PDFs for *R* and approximation to *R*



# Uncertainty evaluation results for *R*

| Method | Best estimate | Standard uncertainty | Endpoints of 95 % coverage interval |
| --- | --- | --- | --- |
| GUF | 13.3 | 19.4 % | 8.3, 18.3 |
| MCM | 13.8 | 20.3 % | 9.5, 19.2 |
| Approx Anal | 13.8 | 19.4 % | 10.1, 18.8 |

Units: $\mu Svh^{-1}$

GUF: GUM uncertainty framework

Adequate agreement between estimates and associated standard uncertainties

Somewhat less between coverage intervals: influence of PDFs obtained for *R*

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# Conclusions

1. GUM uncertainty framework (GUF), MCM
2. Application to measuring neutron dose equivalent rate
3. Model and input quantity characterization using PDFs
4. Difficult quantity: field-specific correction factor $K$
5. Assignment and testing of PDF for $K$
6. Application of GUF and MCM
7. Use of analytic approach for simpler model
8. Adequate agreement for nuclear reactor environments
9. Not necessarily so for other environments
10. MCM generally recommended: accounts for any PDFs